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Introduction: multiobjective optimization and domination

1.1 What is a multiobjective optimization problem ?

An optimization problem is defined as the search for a minimum or a maximum (the optimum) of a function. We can also find optimization problems for which the variables of the function to be optimized are constrained to evolve in a precisely defined area of the search space. In this case, we have a particular kind of optimization called constrained optimization problem.

The need for optimization comes from the necessity of an engineer to give the user a system that fulfills the user's needs. This system must be calibrated so that:

- it occupies the minimum volume needed for its good working (cost of raw materials),
- it uses the least possible energy (working cost),
- it fulfills the user's needs (terms and conditions).

Mathematically speaking, an optimization problem has the following form:

$$\left|\begin{array}{lll} \text{minimize} & f(\overrightarrow{x}) & \text{(function to be optimized)} \\ \text{with} & \overrightarrow{g}(\overrightarrow{x}) \leq 0 & (m \text{ inequality constraints}) \\ \text{and} & \overrightarrow{h}(\overrightarrow{x}) = 0 & (p \text{ equality constraints}) \end{array}\right.$$

We also have $\overrightarrow{x} \in \mathbb{R}^n$, $\overrightarrow{g}(\overrightarrow{x}) \in \mathbb{R}^m$ and $\overrightarrow{h}(\overrightarrow{x}) \in \mathbb{R}^p$. Here, the vectors $\overrightarrow{g}(\overrightarrow{x})$ and $\overrightarrow{h}(\overrightarrow{x})$ represent m inequality constraints and p equality constraints, respectively. This set of constraints delimits a restricted subspace to be searched for the optimal solution.

Usually, we can find two types of inequality constraints:

- Constraints of the type $B_{i_{inf}} \leq x_i \leq B_{i_{sup}}$: values of $\overrightarrow{x}$ which respect these constraints define the "search space". This space is represented in Fig. 1.1a ($n = 2$).
- Constraints of the type $c(\overrightarrow{x}) \leq 0$ or $c(\overrightarrow{x}) \geq 0$: values of $\overrightarrow{x}$ which respect these constraints define the "feasible search space". This space is represented in Fig. 1.1b ($n = 2$).